Adaptive Waveform Design for Cognitive MIMO ISAC

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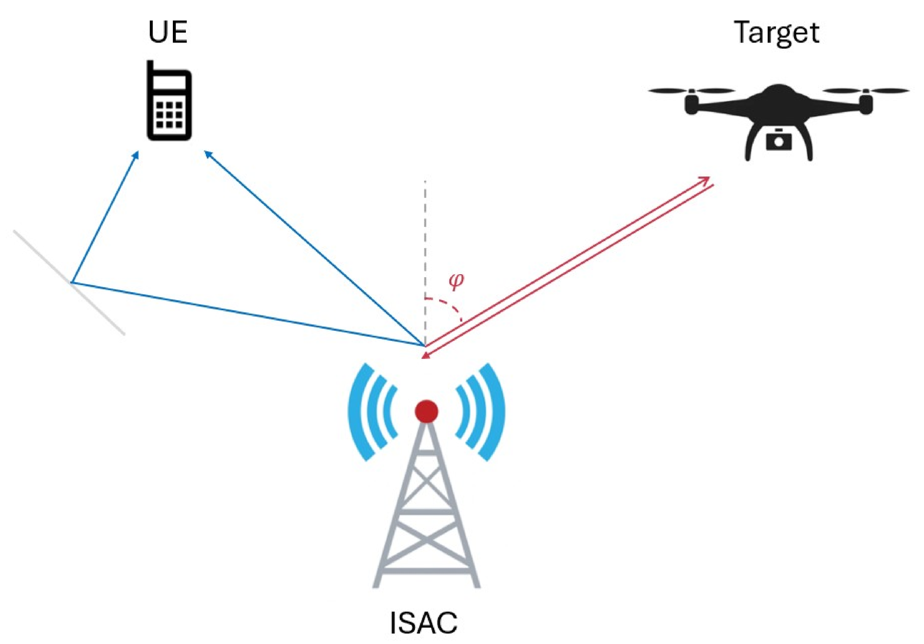
INTRODUCTION

- Integrated sensing and communications (ISAC) approach combines high-throughput communication with precise detection and localization.
- ISAC waveform design is challenging due to joint optimization of communication and sensing criteria.
- The main goal: Cognitive approach for ISAC waveform optimization by sequential waveform design using accumulated information from previous radar observations.
- The cognitive ISAC balancing radar estimation performance with high-throughput communication.

SCENARIO

A collocated MIMO ISAC system simultaneously communicates data to the user and estimates the target DOA:



SIGNAL AND DATA MODELS

ISAC Signal

$$\mathbf{S}_k = \mathbf{R}_{s_k}^{\frac{1}{2}}\mathbf{\Phi}_k, \quad k = 1, 2, \dots .$$

- $\mathbf{\Phi}_k \in \mathbb{C}^{N_t \times L}$ is the data matrix at the k-th step.
- Orthogonal data streams: $\mathbf{\Phi}_k\mathbf{\Phi}_k^H = \mathbf{I}_{N_t}$.
- $\mathbf{R}_{s_k}^{\frac{1}{2}} \in \mathbb{C}^{N_t \times N_t}$ is the beamforming matrix, such that $\mathbf{S}_k\mathbf{S}_k^H = \mathbf{R}_{s_k}$.
- Limited power constraint: $\text{tr}(\mathbf{R}_{s_k}) = P$.

Communication Receiver Model

$$\mathbf{Y}_k = \mathbf{G}_k\mathbf{S}_k + \mathbf{Z}_k\,, \quad k = 1, 2, \dots$$

- Downlink user equipped by N_{r_c} receive antennas.
- $\mathbf{G}_k = \sum_{q=1}^{Q}\beta_{q,k}\mathbf{a}_R(\phi_{a,q})\mathbf{a}_T^T(\phi_{d,q})$ - known multipath communication channel.
- $\mathbf{Z}_k = [\mathbf{z}_{1_k}, \dots, \mathbf{z}_{L_k}]$ - where $\{\mathbf{z}_{l_k}\} \overset{i.i.d.}{\sim} \mathcal{CN}\left(\mathbf{0}_{N_{r_c}}, \sigma_z^2\mathbf{I}_{N_{r_c}}\right)$ represents additive noise.

Radar Receiver Model

$$\mathbf{X}_k = \mathbf{H}(\boldsymbol{\theta}_k)\mathbf{S}_k + \mathbf{V}_k, \quad k = 1, 2, \dots$$

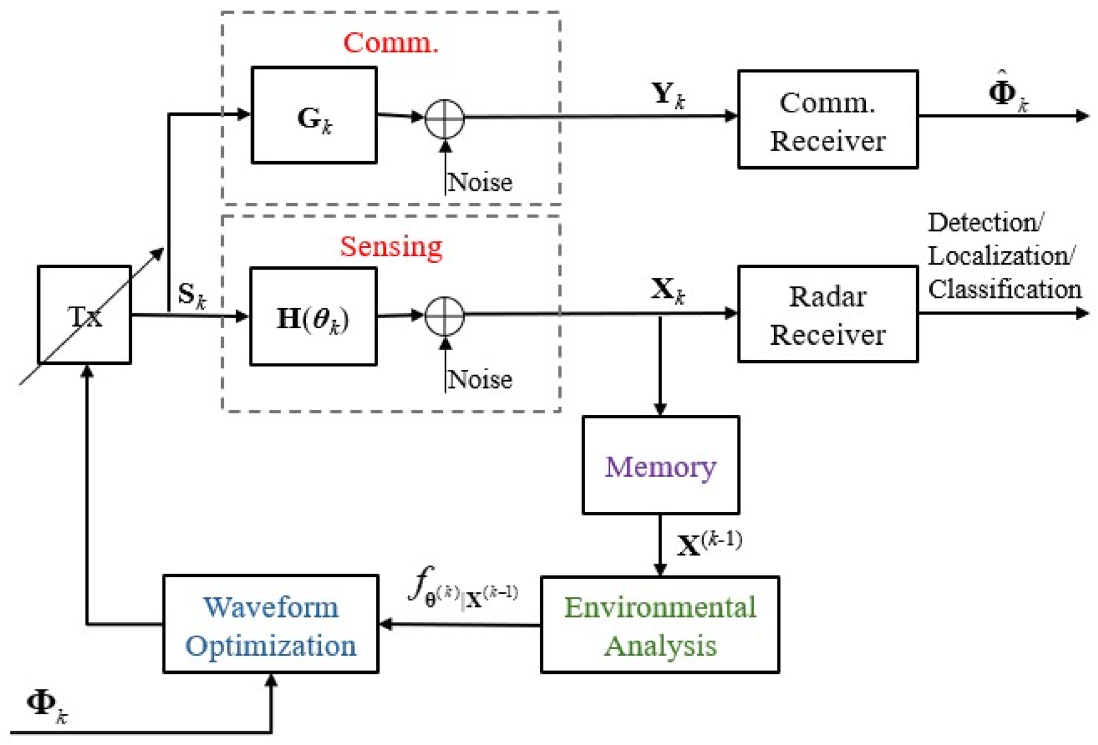
- Radar receiver with N_r antennas.
- $\mathbf{H}(\boldsymbol{\theta}_k) = \alpha_k\mathbf{A}(\varphi)$.
- $\mathbf{A}(\varphi)$ - MIMO steering matrix for target at direction φ.
- $\{\alpha_k\} \overset{i.i.d.}{\sim} N^c\left(0, \sigma_\alpha^2\right)$ - complex amplitudes.
- $\mathbf{V}_k = [\mathbf{v}_{1_k}, \dots, \mathbf{v}_{L_k}]$, where $\{\mathbf{v}_{l_k}\} \overset{i.i.d.}{\sim} N^c\left(\mathbf{0}_{N_r}, \sigma_v^2\mathbf{I}_{N_r}\right)$.
- $\boldsymbol{\theta}_k = [\varphi, \text{Re}\{\alpha_k\}, \text{Im}\{\alpha_k\}]^T$.
- $\boldsymbol{\theta}^{(k)} = [\varphi, \text{Re}\{\alpha_1\}, \text{Im}\{\alpha_1\}, \dots, \text{Re}\{\alpha_k\}, \text{Im}\{\alpha_k\}]^T$.
- $\mathbf{X}^{(k-1)} \triangleq [\mathbf{X}_1, \dots, \mathbf{X}_{k-1}]$ - history of measurements.

Goal: Adaptively design the signal autocorrelation matrix $\mathbf{R}_{s_k}$, such that:

$$\mathbf{R}_{s_k}^{\text{opt}} = \arg\max_{\mathbf{R}_{s_k}} \quad \Psi_r\left(\mathbf{R}_{s_k}, \mathbf{X}^{(k-1)}\right)$$
$$\text{s.t.} \quad \Psi_c\left(\mathbf{R}_{s_k}\right) = \mu_c\,, \quad \mathbf{R}_{s_k} \succeq \mathbf{0}, \quad \text{tr}(\mathbf{R}_{s_k}) = P$$

Ψ_r, Ψ_c - radar and communication performance criteria.

COGNITIVE ISAC WAVEFORM DESIGN



- Radar optimization criterion - conditional Bayesian Cramér-Rao bound (BCRB):

$$\mathbf{BCRB}_{\boldsymbol{\theta}^{(k)}} = \mathbf{J}_{\boldsymbol{\theta}^{(k)}}^{-1} = \text{E}^{-1}\left[-\nabla_{\boldsymbol{\theta}^{(k)}}\left(\nabla_{\boldsymbol{\theta}^{(k)}}\log f_{\boldsymbol{\theta}^{(k)},\mathbf{X}_k|\mathbf{X}^{(k-1)}}\right)^T|\mathbf{X}^{(k-1)}\right]\,,$$

- Communication optimization criterion - channel capacity:

$$C_k = \log_2\left[\det(\mathbf{I}_{N_{r_c}} + \frac{1}{\sigma_z^2}\mathbf{G}_k\mathbf{R}_{s_k}\mathbf{G}_k^H)\right]\,,$$

- Proposed optimization problem:

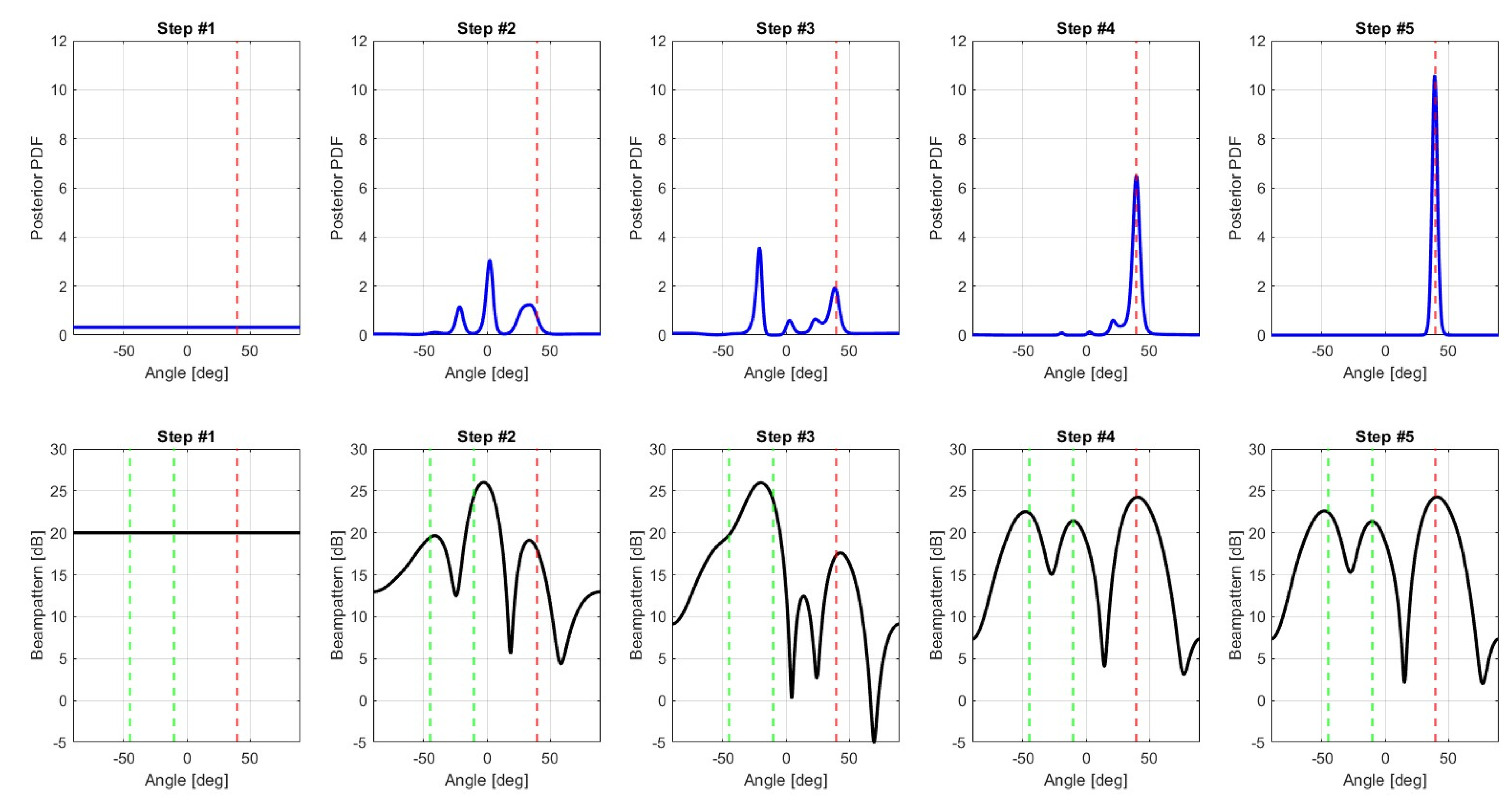
$$\mathbf{R}_{s_k}^{\text{opt}} = \arg\min_{\mathbf{R}_{s_k}} \quad \text{tr}\left(\mathbf{W}_k \cdot \mathbf{BCRB}_{\boldsymbol{\theta}^{(k)}}\right)$$
$$\text{s.t.} \quad C_k \geq \mu_c, \quad \mathbf{R}_{s_k} \succeq \mathbf{0}, \quad \text{tr}(\mathbf{R}_{s_k}) = P$$

- Equivalent convex problem:

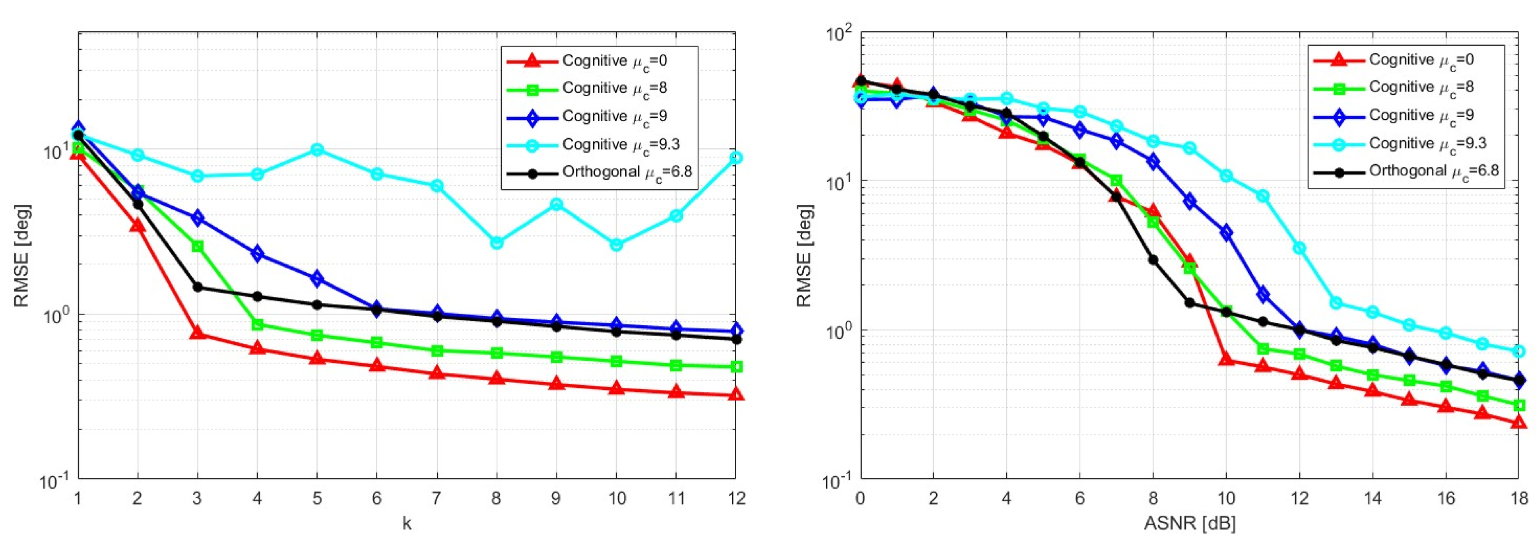
$$\mathbf{R}_{s_k}^{\text{opt}} = \arg\min_{\mathbf{R}_{s_k},\mathbf{t}} \quad \mathbf{w}_k^T\mathbf{t}$$
$$\text{s.t.} \quad \begin{bmatrix}\mathbf{J}_{\boldsymbol{\theta}^{(k)}} & \mathbf{e}_i \\ \mathbf{e}_i^T & t_i\end{bmatrix} \succeq 0, \quad i \in \{1, 2k, 2k+1\}$$
$$C_k \geq \mu_c, \quad \mathbf{R}_{s_k} \succeq \mathbf{0}, \quad \text{tr}(\mathbf{R}_{s_k}) = P$$

SIMULATION RESULTS

- Target DOA (red) - $\varphi = 40°$, channel paths (green) - $\phi_{d,1} = -10°$, $\phi_{d,2} = -45°$.
- $P = 100$, $L = 10$, $N_t = 5$, $N_r = 7$, $N_{r_c} = 5$, $Q = 2$.



Evolution of the posterior PDF (blue) and transmit beamforming (black) over steps.
$\mu_c = 8$bits/sec/Hz, Radar ASNR = 7dB.



Radar ASNR = 11dB

Step $k = 5$

CONCLUSION

- This work proposes a cognitive approach for adaptive ISAC waveform design.
- Simulations demonstrate that this strategy enables efficient power usage for the sensing task by directing the beam toward the likely target direction.
- This efficient method preserves sufficient power for transmission toward the communication channel paths.
- The cognitive ISAC approach enables high estimation performance while maintaining the required communication capacity.